Pergamon
www.elsevier.com/locate/jappmathmech
PII: S0021–8928(03)00099-6
J. Appl. Maths Mechs, Vol. 67, No. 4, pp. 611–620, 2003

0021–8928/$—see front matter

AN INVESTIGATION OF THE IMPACT AND PENETRATION OF SOLIDS OF REVOLUTION INTO SOFT EARTH†

V. G. BAZHENOV, A. M. BRAGOV, V. L. KOTOV and A. V. KOCHETKOV

Nizhnii Novgorod

e-mail: bazhenov@dk.mech.unn.runnet.ru

(*Received 5 February* 2002)

The numerical method proposed earlier in [1] is developed to solve problems of the impact and penetration of rigid and deformable bodies of revolution into soft soil, which are described by Grigoryan's model [2]. The effect of the surface Coulomb friction, the bulk compressibility and the shear strength of soft soil on the forces of resistance and contact pressures in the contact zone is analysed. The results of numerical solutions of problems in a coherent formulation are compared with analytical relations and experimental data on the determination of the forces and coefficients of resistance to the penetration of impactors of different shapes into soft soil.

An approximate solution of problems of the penetration of blunt bodies into soils is well known [3]. The results are based on known representations of the different forms of penetration under supersonic and subsonic conditions and have been confirmed by experimental data. Some experimental data on the magnitude of the forces of resistance to penetration have been presented in [4–8]. A significant number of experimental papers are concerned with analysing the dynamic properties of sandy soil [9, 10], and the equations of state of this soil and other soft soils have been presented in [2, 11]. The difficulties encountered in a theoretical determination of the mechanisms of penetration are primarily due to their unsteady nature, the diversity and inhomogeneity of the properties of natural soils and the fact that the equations of state which reliably describe these properties have not been sufficiently worked out. Analytical methods of solving problems of the penetration of bodies of revolution into soils using simplified representations of the dynamic behaviour of soil have been proposed in [3, 12].

In recent years, numerical methods for investigating the impact and penetration of deformable bodies into soil have been developed using well-known models. The problem of determining the resistance to penetration of a rigid cone in soil without taking account of the shear properties of the soil has been solved [14] using a Godunov numerical scheme [13], and a technique for calculating the interaction of structural elements with soil media within the framework of a model of a plastic gas has been proposed in [15, 16]. This technique has been used for the numerical investigation of the penetration of an undeformable impactor into an argillaceous medium and snow [7] and the use of a scheme of the first order of accuracy for the numerical modelling of the behaviour of elastoplastic media (soils which are described by Grigoryan's model [2]) has been considered in [1, 17, 18].

An experimental–theoretical investigation of the processes when impactors with plane, hemispherical and conical caps interact with sandy soil over a range of variation of the impact velocities from 100 m/s up to 500 m/s is presented below. It is established that the choice of the diagram for the bulk compressibility of soft soil mainly affects the maximum value of the resistance to the penetration of the impactor with a plane end. It is essential to take account of the shear strength of the soil when modelling the penetration of impactors with hemispherical and conical caps and, also, at the stage of developed penetration. The effect of sliding friction on the resistance when conical impactors penetrate into soil is determined by the semi-angle of the cone. The values of the resistance forces at the transient stage and at the stage of developed penetration, obtained using well-known analytical relations, are in satisfactory agreement with the numerical and experimental data in the range of velocities above 300 m/s. The proposed modification of the analytical method, which involves the use of the solution of a problem of the decay of a discontinuity in order to find the density in the shock wave, enables satisfactory agreement to be obtained between the analytical solution and the numerical and experimental results over a wider range of impact velocities.

†*Prikl. Mat. Mekh.* Vol. 67, No. 4, pp. 686–697, 2003.

1. MATHEMATICAL FORMULATION OF THE PROBLEM

The axisymmetric problem of the impact and penetration of a body of revolution into soil is formulated in a cylindrical system of coordinates $xy\theta$, where x is the axis of revolution and $y > 0$. We will represent the system of equations [2], which describes the dynamics of the soil, on eliminating the dependence of the unknown quantities on θ, in the form

$$\begin{aligned}
&(\rho)_{,t} + (\rho u)_{,x} + (\rho v)_{,y} = -(\rho v)/y \\
&(\rho u)_{,t} + (\rho u^2 + p - s_{xx})_{,x} + (\rho uv - s_{xy})_{,y} = -(\rho uv - s_{xy})/y \\
&(\rho v)_{,t} + (\rho uv - s_{xy})_{,x} + (\rho v^2 + p - s_{yy})_{,y} = -(\rho v^2 - 2s_{yy} - s_{xx})/y \\
&D_J s_{xx} + \lambda s_{xx} = 2G(2u_{,x} - v_{,y} - v/y)/3 \\
&D_J s_{xy} + \lambda s_{xy} = G(u_{,y} + v_{,x}) \\
&D_J s_{yy} + \lambda s_{yy} = 2G(2v_{,y} - u_{,x} - v/y)/3
\end{aligned} \tag{1.1}$$

The notation adopted is as follows: t is the time, ρ is the density, s_{ij} are the components of the stress deviator tensor, $(i, j = x, y)$, D_J is the Jaumann derivative with respect to time and G is the shear modulus. A subscript after a comma denotes differentiation with respect to the corresponding variable. The parameter λ can take the values $\lambda = 0$ in the case of elastic deformation and $\lambda > 0$ if a condition of plasticity occurs. In this technique, a procedure for normalizing the components of the deviator tensor to the quantity $1/\sqrt{\lambda}$ is used, which is equivalent to the complete relations in the theory of plastic flow.

The criterion of shear plasticity in the case of a soil medium in defined in the form of a function of the second invariant of the stress tensor of the pressure p

$$\frac{1}{2} s_{ij} s^{ij} = F(p) \tag{1.2}$$

(summation is carried out over repeated subscripts).

The relations between the pressure and the density are taken in the form

$$p = \begin{cases} f_1(\rho), & dp/dt > 0 \\ f_2(\rho, \rho^*), & dp/dt \le 0 \end{cases} \tag{1.3}$$

The first of these equations is the shock adiabatic curve and the second gives the curve for the stress relief against the maximum density ρ^* which is attained during active loading of the soil. In the case of active loading, it is assumed that $\rho^* = \rho$ and that the stress relief and secondary additional loading are described by the equation $d\rho^*/dt = 0$, where d/dt is the total time derivative. It is more convenient to write this equation in Euler variables and in a divergent form by transforming it using the law of conservation of mass to the form of a law of conservation for $\rho\rho^*$, as has been done previously in [14],

$$(\rho\rho^*)_{,t} + (\rho\rho^* u)_{,x} + (\rho\rho^* v)_{,y} = -(\rho\rho^* v)/y$$

An explicit difference scheme [1, 19] of the first order accuracy was used to solve the problems. This scheme combines the Lagrange and Euler approaches to the description of the motion of a compressible medium. The following boundary conditions are employed: "*free surface*" $\sigma = 0$, $\tau = 0$, where σ and τ are the normal and tangential components of the stresses in the free boundary, "*impermeability along the normal*": $v_{n1} = v_{n2}$ and "*slippage with Coulomb friction*": $\tau = k\sigma_n$ in the tangential direction. Here, v_n is the velocity component along the normal to the surface of contact and k is the coefficient of Coulomb friction. The conditions on the surfaces of contact of the bodies and the media with different physical and mechanical properties also take account of the phenomenon of detachment, the formation of free surfaces (cavitational hollows) and their possible subsequent collapse. They are formulated as a combination of the impermeability conditions on those segments of the surfaces which find themselves in contact at a given instant of time and the conditions on the free boundaries in the remaining segments. The inequality $q < q_k$, where q is the contact pressure and q_k is a certain constant which characterizes the resistance to detachment, serves as the criterion of the transition from conditions of impermeability to the condition in the free surface (detachment). The geometrical intersection of the free surfaces of the bodies is the criterion of coming into contact.

The numerical implementation of the contact conditions is based on the separation and tracking during calculations of the contacting and free surfaces. The contact forces are determined by the simultaneous solution of the difference equations for the motion of the structural elements and the dynamic compatibility relations for the waves in the contacting media. Effective algorithms have been developed [19] which implement the contact conditions (including when the contacting surfaces are approximated by the meshes with non-coincident mesh points) for problems of the collision and penetration of deformable and rigid bodies into compressible media.

2. THE EQUATION OF STATE OF SOIL

The technique which has been developed enables us to describe the interaction of rigid and deformable bodies and structural elements with different soft soils if the functions (1.2) and (1.3) are known. In this paper, the experimentally obtained values of the constants and auxiliary relations are presented for sand with natural moisture content. The decisive relations (1.3) between the bulk deformation $\varepsilon = 1 - \rho_0/\rho$ and the pressure p in sandy soil when the pressure changes up to 10 MPa are based on the experimental results in [9] on explosions of spherical charges in sandy soil and are taken in the form of the relation

$$p = M\varepsilon^n \tag{2.1}$$

The shock adiabatic curve, obtained using the results of the plane wave shock experiments described in [10], is used at pressures greater than 250 MPa. The linear relation $D = A + BU$ between the shock wave velocity and D the mass velocity behind the wave from U is transformed, using the Hugoniot conditions, to the form

$$p = \rho_0 A^2 \varepsilon (1 - B\varepsilon)^{-2} \tag{2.2}$$

where ρ_0 is the initial density of the soil and A and B are constants. Within the range from 10–250 MPa, an interpolating, parametric, cubic Bezier polynomial [18] is used which ensures the continuity of the speeds of sound (of the derivative $dp/d\rho$) at the junction points.

$$\mathbf{r}(w) = \{\rho(w), p(w)\} = (1-w)^3\mathbf{r}_1 + 3w(1-w)^2\mathbf{r}_2 + 3w^2(1-w)\mathbf{r}_3 + w^3\mathbf{r}_4 \tag{2.3}$$

The polynomial (2.3) in (ρ, p) coordinates is characterized by the fact that, when the parameter w changes from 0 to 1, it passes through the points (ρ_1, p_1) and (ρ_4, p_4), and the tangent at these points coincides with the straight lines passing through the points (ρ_1, p_1), (ρ_2, p_2) and (ρ_3, p_3), (ρ_4, p_4) respectively. At the same time,

$$\rho_2 = 1 + \alpha \cdot \rho_1, \quad \rho_3 = 1 - \beta \cdot \rho_4 \tag{2.4}$$

and the corresponding pressures are found by substituting the values of the density into the equations of the tangents. The equations of the tangents and the values of the polynomial at the reference points (ρ_1, p_1) and (ρ_4, p_4) are calculated in accordance with diagrams (2.1) and (2.2)

The loading diagram is shown in Fig. 1, where relation (2.1) is represented by the solid curve 1, curve 2 is the shock adiabatic curve in the form of (2.2), and the approximating polynomial (2.3) is represented by the dashed curve. The stress relief of the medium is described by a two-segment, dashed line [18] (curve 3). The dependence of the yield point on the pressure is assumed to be linear [10]

$$F(p) = \frac{1}{3}(2\alpha p + b)^2 \tag{2.5}$$

where α is the tangent of the angle of internal friction and b is the adhesion.

The equation of state of sand, which has been presented, enables us to describe the unsteady deformation of the sand over a wide range of load variations. This equation has been used earlier to investigate explosive action on sandy oil [18] and shock and wave interaction in a system of split Hopkinson rods [20].

3. FORMULATION OF THE PROBLEMS AND BASIC RESULTS

Below, we presented the results of calculations of the impact and penetration into sand of cylindrical steel impactors with a diameter of 20 mm and the following mechanical properties of the impactor

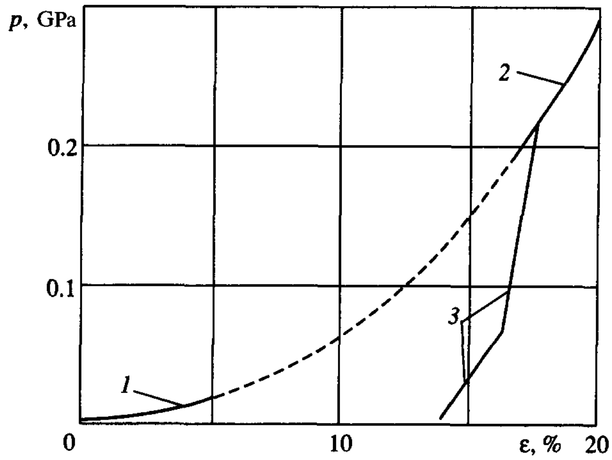

Fig. 1

material: Young's modulus 200 GPa, Poisson's ratio 0.3, density 7.8 g/cm^3 and yield point 1.8 GPa. For sand with a density $\rho_0 = 1.76$ g/cm^3, the constants in the power relation (2.1) were taken as: $M = 2.1$ GPa and $n = 1.8$ [9], and the constants of the shock adiabatic curve of (2.2) were $A = 500$ m/s and $B = 2.4$ [10]. The values of the constants in the interpolating polynomial were: $\alpha = \beta = 0.06$, $\rho_1 = 1.86$ g/cm^3 and $\rho_4 = 2.15$ g/cm^3. The parameters of the functional relation for describing the stress relief were: $\gamma_c = 2$, $\gamma_p = 5$ (this notation corresponds to that adopted earlier in [11]), the initial speed of sound during stress relief $c_0 = 350$ m/s, the shear modulus $G = 150$ MPa and the constants in relation (2.5) were $\alpha = 0.6$, $b = 0$ [9].

The interaction of an extended impactor with a plane end with sandy soil. We will now present the numerical results of an investigation into the collision, formation of a pressure pulse and its propagation in a rod-like impactor when it is impacted by a container with sand at a speed of 276 m/s. The formulation of the problems corresponds to the formulation of the inverse experiment [8].

The results of the calculations suggest that the initial stage in the interaction of the impactor with the soil is accompanied by the formation of a cavity which confirms similar conclusions drawn earlier in [3, 8]. The magnitude of the calculated integral contact force, which is compared with the experimental results depicted by the small open circles, is shown by the solid curve in Fig. 2. The ratio of the maximum value of the forces to their values at the quasi-steady stage of penetration is equal to approximately 2. The maximum values of the contact forces in the case of the impact of rigid and elastically deformable impactors (yield point 1.8 GPa) on soil do not differ by more than 5%. The formation of plastic deformations in rod-like striker with a yield point of 0.3 GPa (the dot–dash curve in Fig. 2) leads to a significant reduction in the value of the resistance force, calculated using the longitudinal stresses in the rod, and to appreciable errors in determining the contact forces.

The experimental curves of the maximum values of the resistance force (the black dots) and of the forces at the quasi-steady stage of penetration (the small open circles) shown in Fig. 3. The maxima of the resistance forces, calculated using a well-known method [3], are represented by the solid curve 1, and the results of the numerical calculations are represented by the dashed line 1,

In order to estimate the forces at the quasi-steady stage, we use the results obtained earlier in [3] for the subsonic motion of a body at a constant rate of penetration

$$F = C_x(1 + 4\alpha l/d)S\rho_0 V^2/2 \tag{3.1}$$

Here l is the distance from the centre section to the leading point, that is, the vertex of the body surface and d is the diameter of the cross-section of the body. In the case of quasi-steady motion of a cylinder with a flat end, a region of compacted soil is formed which moves together with the body. The dimensions of this compacted region have to be taken as the parameters l and d in formula (3.1). If it is assumed that the compacted core has the form of a hemisphere or an ogival, then $l = 1$ and $d = 2$ cm. The remaining parameters are defined in the following way: $S = \pi d^2/4$ and V is the penetration velocity of the body. The solid curve 2 corresponds to the results obtained using formula (3.1) when $C_x = 1$ and

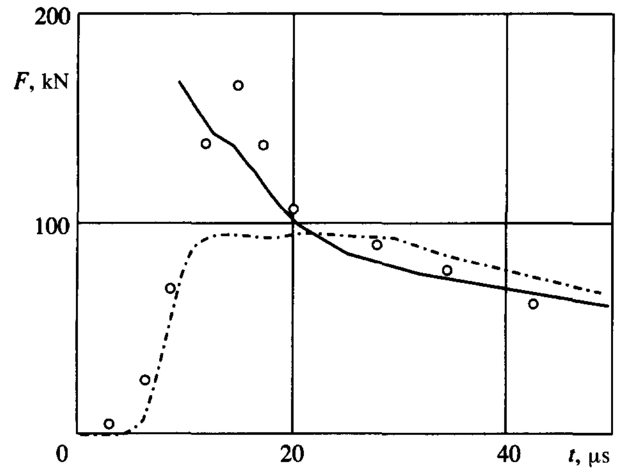

Fig. 2

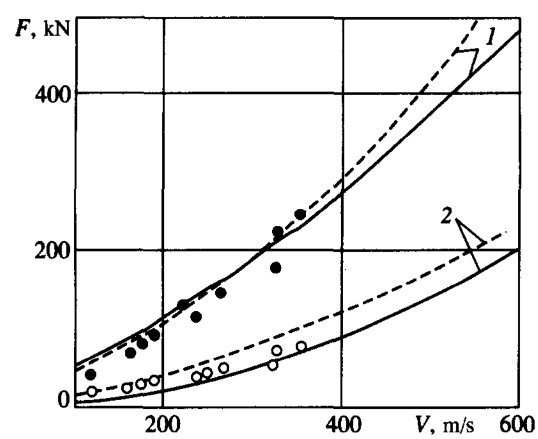

Fig. 3

the dashed curve 2 corresponds to the numerical calculations. Over a range of impact velocities of 100–600 m/s, the difference in the forces at the quasi-steady stage of penetration obtained using experimental data and numerically does not exceed 15%.

In this problem, the shear strength and the irreversibility of the bulk deformations of the soil have practically no effect on the maximum values of the longitudinal stresses in the rod-like impactor. The difference only manifests itself with the passage of time at the stage of the developed quasi-steady penetration and can reach a value of 100% [8]. The effect of allowing for the forces of sliding friction of the soil on the impactor surface is small, which is explained by the small radial displacement of the soil on the end of the impactor and the fact that there is no interaction with the lateral surface of the rod when the flow around it is cavitational in character.

Interaction of a hemispherical impactor with soil. A numerical calculation of the interaction of a hemispherical impactor with soil has also been carried out in a formulation which corresponds to the inverse experiment [8]. It is assumed that the penetration occurs at a constant rate.

The experimental data, that is, the maximum values of the resistance force to penetration, are shown as a function of the velocity of impact of the hemispherical impactor by the small open circles in Fig. 4 and the results of the numerical calculations are represented by the dashed curve. It is seen that there is satisfactory agreement between the numerical and experimental results over the range of velocities 150, ... , 400 m/s and the disagreement lies within the limits of experimental error (10–20%).

In the case of the subsonic motion of a body with a hemispherical cap in soil, the values of the forces at the quasi-steady stage also estimated by formula (3.1). In experiments and numerical calculations,

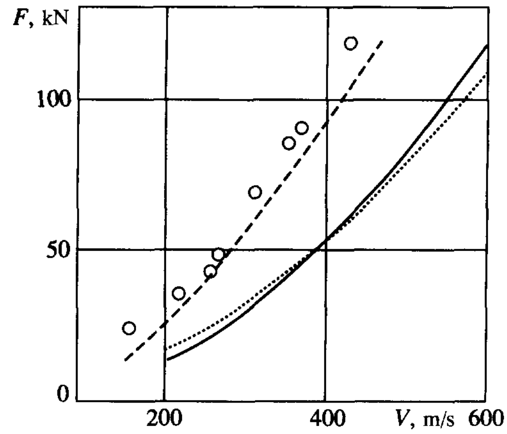

Fig. 4

a flow separation angle of 60° to 70° was observed. For this value of the angle, the distance from the centre section to the vertex of the body is equal to 0.5, d= 1.73 cm. The forces at the quasi-steady stage of penetration, obtained using formula (3.1) with $C_x = 1$ are shown by the solid curve in Fig. 4 and the results of the numerical calculations are represented by the dotted curve. The good agreement between the numerical and the analytical results confirms the applicability of the formulae [3] for estimating the parameters of the quasi-steady stage of the motion of hemispherical impactors.

It is well known that, in the case of the motion of blunt bodies in soil, neglect of the shear strength leads to a reduction in the resistance by a factor of $1/(1 + 4\alpha/d) \sim 0.6$ [3]. The assumption that the soil has no shear properties in the numerical calculations leads to a reduction in the resistance by a factor of approximately 0.5–0.6, which is also confirmed by the experimental data [8]. Unlike the case of the penetration of an impactor with a plane end, when account is taken of friction in the numerical calculations, the maximum value of the resistance increases by 15 to 20%. The value of the forces at the quasi-steady stage of penetration increase by 5 to 10%. When the velocity of penetration increases (greater than 400 m/s), the effect of sliding friction on the resistance decreases.

Penetration of a cylindrical impactor with a conical cap into soil. A sketch of the problem of the vertical penetration of a cone, with an arbitrary vertex angle 2β, into soil is shown in Fig. 5. The transient phenomena: the impact on the soil surface and the subsequent initial period of penetration into the soil, are primarily considered.

The following assumptions, under which the problem of the penetration of a cone of finite span admits of an analytical solution, have been previously introduced in [12]:

(1) the soil particles move along a normal to the surface of the penetrating cone;

(2) the density retains the maximum value which has been attained behind the front which is formed in the soil during the penetration of the shock wave;

(3) the pressure distribution along the contact surface is constant when x changes from 0 to $h\cos^2\beta$ and increases non-linearly when $x \in [h\cos^2\beta, h]$.

Here, the change in x is measured in the direction from the vertex of the penetrating cone, h is the distance from the vertex to the free surface of the soil (the depth of penetration) and β is half the aperture angle of the cone.

A numerical analysis of the penetration of a conical impactor into study soil was carried out in order to check these hypotheses.

Different versions of the penetration of a cone with a mass of 44 g, taking account of the drop in the velocity and penetration of the cone into the soil, were calculated for a constant velocity V = 225 m/s, R = 1 cm and 2β = 60°. In the first series of calculations, no account was taken of the sliding friction of the particles on the surface of the cone and, in the second series, friction was taken into account using Coulomb's law. In the case when there is no friction, the soil particles moved practically along the normal to the contact surface. The existence of friction leads to a deviation of the trajectories of the particles along the direction of motion of the penetrating cone. Sliding friction was taken into account in the subsequent calculations. The value of the friction coefficient remained constant during a calculation and was equal to 0.3 [18].

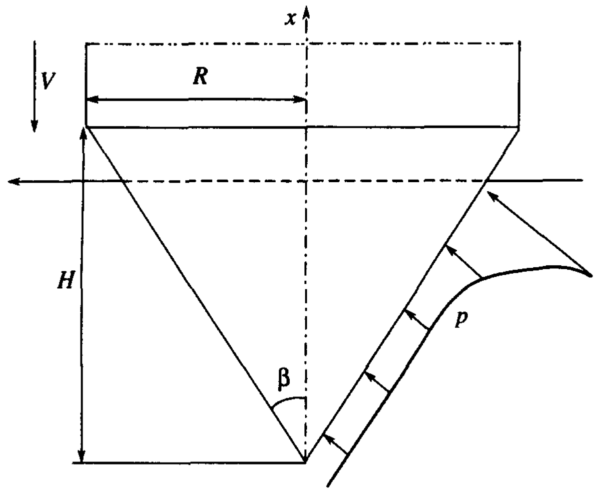

Fig. 5

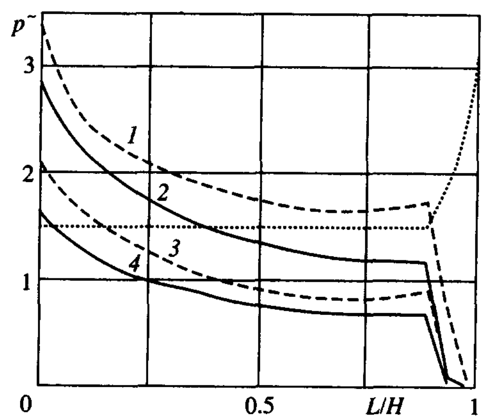

Fig. 6

Numerous experiments have shown that the stress relief in a real soil is close to vertical only in the case of high pressures of the order of 1 GPa. When the loading is reduced, there is also a substantial reduction in the stress relief modulus. The theoretical prerequisite for the conservation of the maximum value attained by the density corresponds to an infinite (or quite high) velocity of the stress relief wave in the soil. The effect of the magnitude of the stress relief modulus on the parameters of the penetration process was analysed in the following manner. Different versions were calculated in which the velocity of the stress relief wave c_0 at a low initial pressure [18] was equal to 200 and 2000 m/s.

The results of the comparison are shown in Fig. 6 in the form of the distribution of the dimensionless contact pressure $p^{\sim} = p/(\rho V^2/2)$ along the generatrix of the cone. The results of calculations with an initial stress relief velocity of 200 m/s are shown by the solid curves and the results of calculation when $c_0 = 2000$ m/s are shown by the dashed curves. Curves 1 and 2 correspond to the results of calculations of the penetration of a cone with a constant velocity $V = 225$ m/s. The results of a calculation of the penetration of a cone with a mass of 44 g are shown by curves 3 and *4*. The curves are presented at the instant of time $2t^*$, ($t^* = H/V$). It is clear that the effect of stress relief on the contact pressure distribution appears to a greater extent in the motion with constant velocity (curves 1 and 2). In the case of the chosen parameters (aperture angle, velocity of penetration and type of soil), the maximum value is reached in the neighbourhood of the cone vertex. The stress relieving action of the free surface is propagated in a narrow layer close to the surface. Hence, the suggestion put forward earlier [12] concerning a maximum of the contact pressure at the intersection of a cone with a free surface, which is shown by the dotted curve in Fig. 6, turns out to be incorrect. A constant value of the dimensionless contact pressure is obtained using the formula [12]

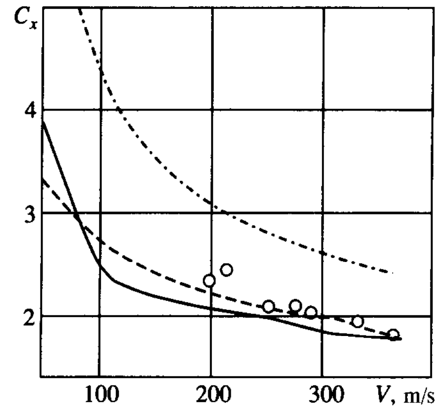

Fig. 7

$$p = \frac{2\sin^2\beta}{b}\left[ba^{\nu/2} - \frac{a^{\nu/2-1}-1}{\nu-2} + \frac{a^{\nu/2}-1}{\nu}\right]S \tag{3.2}$$

where $b = \frac{\rho_0}{\rho}$, ρ is the density on the shock wave, $a = \frac{1}{1-b}$, $\nu = \frac{2\sin\varphi}{1+\sin\varphi}$, $S = \pi h^2 \operatorname{tg}^2\beta$, and the angle of internal friction $\varphi = 30°$.

Earlier, in [12], the pressure in the shock wave was calculated using the formula

$$p = \rho_0 \operatorname{tg}^2\beta V^2 \tag{3.3}$$

and later, using an equation of state for the soil of the type of (1.3), the corresponding density, which is used in expression (3.2), was calculated. The coefficient of resistance was defined as the ratio of the pressure (3.2) to the cross-section area of the cone. In the numerical solutions of the problem, the value of the coefficient of resistance was calculated using the formula

$$C_x = F(t)\left(\frac{1}{2}\rho_0 V^2(t)\pi r^2(t)\right)^{-1} \tag{3.4}$$

where $F(t)$ is the numerical value of the resistance to penetration of the cone into the soil and r is the radius of the cross-section of the depressed part of the cone. In the case of penetration to a depth H and further, the cross-section area remains constant $r(t)$ = const = R.

Graphs of the coefficient of resistance to penetration against the velocity of penetration are shown in Fig. 7. The analytical solutions, obtained using formulae (3.2) and (3.3), are represented by the dot–dash curve, the results of the numerical calculations by the solid curve and the results of the inverse experiment by the small open circles. For the whole range of impact velocities investigated, the numerical and analytical [12] results are significantly different, particularly in the case of impact velocities below 100 m/s.

The analytical solution can be refined if the density at the shock wave front ρ is determined taking account of the non-linearity of the soil compression diagram, obtained from the solution of the problem of the decay of an arbitrary discontinuity [21]. The value of the velocity U of the contact discontinuity is determined from the equation

$$(U-u_0)^2 = (\sigma-\sigma_0)(1/\rho - 1/\rho_0) \tag{3.5}$$

where u_0 is the velocity and σ_0 is the stress before the wave front. At the initial instant of penetration $u_0 = \sigma_0 = 0$. If it is assumed that the soil particles move along the normal to the surface of the penetrating cone, the velocity of the contact discontinuity is given by the expression $U = V\sin^2\beta$ and Eq. (3.5) is transformed to the form

$$\sigma(\rho)(1/\rho - 1/\rho_0) = V^2\sin^2\beta \tag{3.6}$$

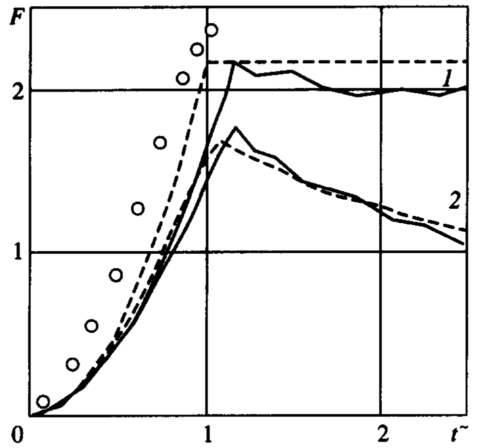

Fig. 8

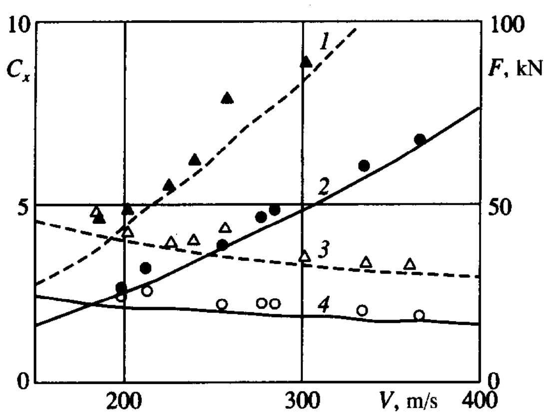

Fig. 9

When the relation between the stress and the density is known

$$\sigma(\rho) = -p(\rho) - \frac{4}{3}G\ln\frac{\rho}{\rho_0}$$

the value of the density can be calculated from Eq. (3.5) by Newton's method, taking account of the dependence of the yield point on the pressure (2.5). With this method of calculating the coefficient of resistance, the analytical results (the dashed curve in Fig. 7) are in better agreement with the numerical and experimental data.

Graphs of the dimensionless resistance to penetration against the dimensionless time (the values of the force are divided by $1/2\rho V^2\pi R^2$ and the time is divided by t^*) are shown in Fig. 8. The notation used in this figure is the same as the employed in Fig. 7. Curves 1 correspond to a constant velocity of penetration and curves 2 were obtained taking account of the fall in the velocity accompanying the motion of a body with a mass of 44 g. Calculations carried out without taking account of sliding friction led to a value of the force which was reduced by a factor of $(1 + \text{ctg}\beta)k_{mp} \approx 1.5$.

A comparison of the results of the numerical calculations of the maximum force F (curves 1 and 2), the coefficient of resistance C_x (curves 3 and 4) and experiments plotted against the velocity of impact V over a range of 150 to 400 m/s is shown in Fig. 9 for cones with an aperture angle $2\beta = 60°$ (the calculated data are shown by the solid curves and the experimental data by the small open and dark circles) and for a cone with an aperture angle of 100° (the calculated data are represented by the dashed curves and the experimental data by the small open and dark triangles). Curves 1 and 3 were constructed for the following values of the parameters: $2\beta = 100°$, and curves 2 and 4 when $2\beta = 60°$. For the experimental data, the small open circles represent the coefficient of resistance, and the small dark circles show the resistance force to penetration of a cone with an aperture angle of $2\beta = 60°$, the small open triangles represent the coefficient of resistance and the small dark triangles show the resistance force to penetration of a cone with an aperture angle of $2\beta = 100°$.

We wish to thank A. K. Lumonov and V. V. Balandin for carrying out the experiments, and also S. V. Krylov for his help in carrying out the calculations and discussing the results.

This research was carried out within the framework of the "State Support for Leading Scientific Schools" Programme (NSh – 1136.2003.8) and with the support of the Russian Foundation for Basic Research (01-01-00556, 02-01-00594) and the Ministry of Education of the Russian Federation (RD02-2.10-4, E00-4.0-85).

REFERENCES

1. ABUZYAROV, M. K., BAZHENOV, V. G., KOTOV, V. L., KOCHETKOV, A. V., KRYLOV, S. V. and FEL'DGUN, V. R., A method for the decomposition of discontinuities in the dynamics of elastoplastic media. *Zh. Vychisl. Mat. Mat. Fiz.*, 2000, **40**, 6, 940–953.
2. GRIGORYAN, S. S., Basic representations of the dynamics of soils. *Prikl. Mat. Mekh.*, 1960, **24**, 6, 1057–1072.
3. GRIGORYAN, S. S., Approximate solution of the problem of the penetration of a body into soil. *Izv. Akad. Nauk SSSR. MZhG*, 1993, 4, 18–24.
4. ZLATIN, N. A., KRASIL'SHCHIKOV, A. P., MISHIN, G. I. and POPOV, N. N., *Ballistic Equipment and its Use in Experimental Investigations*, Edited by N. A. Zlatin and G. I. Mishin. Nauka, Moscow, 1974.
5. ALLEN, W. A., MAYFIELD, E. B. and MORRISON, H. L., Dynamic of a projectile penetrating sand. *J. Appl. Phys.*, 1957, 28, 125–137.
6. FORRESTAL, M. J., LEE, L. M., JENRETTE, B. D. and SEATCELL, R. E., Gas-gun experiments determine forces on penetrators into geological targets. *Trans. ASME J. Appl. Mech.*, 1984, **51**, 3, 602–607.
7. BUKHAREV, Yu. N., GANDURIN, V. P., KORABLEV, A. E., MOROVOV, V. A. and KHAIMOVICH, M. I., Experimental investigation of the penetration of a non-deformable impactor into an argillaceous medium and snow. In *Applied Problems of Strength and Plasticity. Analysis and Optimization of Structures*. Izd. Nizhegorod. Univ., Nizhnii Novgorod, 1991, 48, 99–106.
8. BAZHENOV, V. K., KOTOV, V. L., KRYLOV, S. V., BALANDIN, V. V., BRAGOV, A. M. and TSVETSKOVA, Ye. V., Experimental-theoretical analysis of the unsteady interaction of deformable penetrators with a soil medium. *Zh. Prikl. Mekh. Tekh. Fiz.*, 2001, **42**, 6, 190–198.
9. RYKOV, G. V., Experimental investigation of the stress field accompanying an explosion in sandy soil, *Zh. Prikl. Mekh. Tekh. Fiz.*, 1964, 1, 85–89.
10. LAGUNOV, V. A. and STEPANOV, V. A., Measurement of the dynamic compressibility of sand at high pressure. *Zh. Prikl. Mekh. Tekh. Fiz.*, 1963, 1, 88–96.
11. VOVK, A. A., ZAMYSHLYAYEV, B. V., YEVTEREV, L. C. *et al.*, *Behaviour of Soils under the Action of Pulsed Loads*. Naukova Dumka, Kiev, 1984.
12. SAGOMONYAN, A. Ya., *Penetration*. Izd. Mosk. Gos. Univ. Moscow, 1974.
13. GODUNOV, S. K. (Editor), *The Numerical Solution of Multidimensional Problems of Gas Dynamics*. Mir. Moscow, 1976.
14. KIRILENKO, G. A. and SAGOMONYAN, A. Ya., Numerical modelling of penetration into soil. *Izv. Akad. Nauk ArmSSR. Mekhanika*, 1986, **39**, 1, 47–51.
15. BAZHENOV, V. G., KOCHETKOV, A. V. and KRYLOV, S. V., Analysis of non-linear effects accompanying the high velocity penetration of bodies into a compressible fluid. *Prikl. Mekh.*, 1986, **22**, 2, 125–127.
16. BAZHENOV, V. G., KOCHETKOV, A. V., KRYLOV, S. V. and UGODCHIKOV, A. G., High velocity impact of elastoplastic thin-walled structures on the surface of a compressible fluid. *Izv. Akad. Nauk SSSR. MTT*, 1984, 5, 162–169.
17. BAZHENOV, V. G., KOZLOV, Ye. A. and KRYLOV, S. V., Numerical modelling of two-dimensional non-linear problems of the shock interaction of deformable media and structures using Godunovs method. In *Applied Problems of Strength and Plasticity. Investigation and Optimization of Constructions*. Izd. Gor'k. Univ., Gor'kii, 1990, 45, 99–106.
18. BAZHENOV, V. G., KOTOV, V. L., KOCHETKOV, A. V., KRYLOV, S. V. and FLE'DGUN, V. R., Investigation of wave processes in a soil medium accompanying the explosion of a laid charge, *Izv. Ross Akad. Nauk. MTT*, 2001, 2, 70–77.
19. BAZHENOV, V. G., ZEFIROV, S. V., KOCHETKOV, A. V., KRYLOV, S. V. and FEL'DGUN, V. R., The "Dynamics-2" software package for solving plane and axisymmetric non-linear problems of the unsteady interaction of structures with compressible media. *Mat. Modelirovaniye*, 2000, **12**, 6, 67–72.
20. BAZHENOV, V. G., BRAGOV, A. M., KOTOV, V. L., ZEFIROV, S. V., KOCHETKOV, A. V., KRYLOV, S. V. and LOMUNOV, A. K., Analysis of the applicability of the modified Kol'skii method to dynamic experiments on soil media in a deformable ring. *Zh. Prikl. Mekh. Tekh. Fiz.*, 2000, **41**, 3, 155–162.
21. KOTOV, V. L., Solution of a non-linear problem of the decay of the wave discontinuities in soft soil media. *Vestnik Nizhegorod. Univ., Ser. Mekhanika*, 2000, 1, 135–144.

Translated by E.L.S.